Lagrangian modeling and visualization of rosette outfall plumes

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ABSTRACT: A recently-developed Lagrangian jet model (JETLAG) is combined with visualization technology to give virtual reality simulations of three-dimensional rosette-shaped jet groups issuing from ocean outfalls. The main features of the new entrainment formulation in the Lagrangian model, in particular the prediction of mixing in the weak-strong current transition, and the coflowing jet, are described. Representative comparisions of model predictions with experimental data are presented. The incorporation of computer graphic techniques to develop a PC-based, interactive flow visualization tool is summarized. The resulting model, VISJET, portrays the evolution and interaction of multiple buoyant jets discharged at different angles to the ambient tidal current. The model enables the determination of the degree of merging of the interacting plumes, and is capable of communicating the predicted environmental impact effectively to the stakeholders.

INTRODUCTION

For environmental impact assessment and outfall design studies, it is desirable to have a computer model that is able to i) predict the initial mixing of buoyant wastewater discharges in a current, and ii) communicate the predicted impact effectively to the user or stakeholder. For discharges into many coastal waters, water quality objectives simply cannot be achieved if the effect of a tidal current (which is present for most of the time) is not taken into account. However, the prediction of tracer concentration (or dilution) along the unknown jet trajectory is a complicated problem which is not fully resolved. In particular, few models can treat satisfactorily jets with three-dimensional trajectories, such as oblique buoyant jets or rosette-shaped jet groups discharging from modern ocean outfall risers. For risk analysis and post-operation monitoring especially, it is necessary to have a robust and accurate model that is capable of giving predictions under the whole range of jet orientation, ambient current and stratification conditions. As far as we are aware, such a model has hitherto not been reported. Further, the use of visualization technology in enhancing the effective and accurate communication of the predicted impact to the user has not been explored in any depth.

This paper reports recent work on the development of a PC-based flow visualization computer model developed for ocean outfall discharges. The model combines a Lagrangian model, JETLAG, which has undergone significant developments in recent years, with computer graphics techniques to give virtual reality simulations of merging jets issuing from an ocean outfall. While the jet/plume in a crossflow is a well-researched subject in environmental hydraulics; there are nonetheless many outstanding technical issues. In the following a new Lagrangian entrainment formulation is first presented. Representative comparisons of model predictions with experimental data are then presented for flow situations that are difficult to model but often encountered in practice. The use of computer graphics for visualizing and assisting the model predictions is then highlighted.

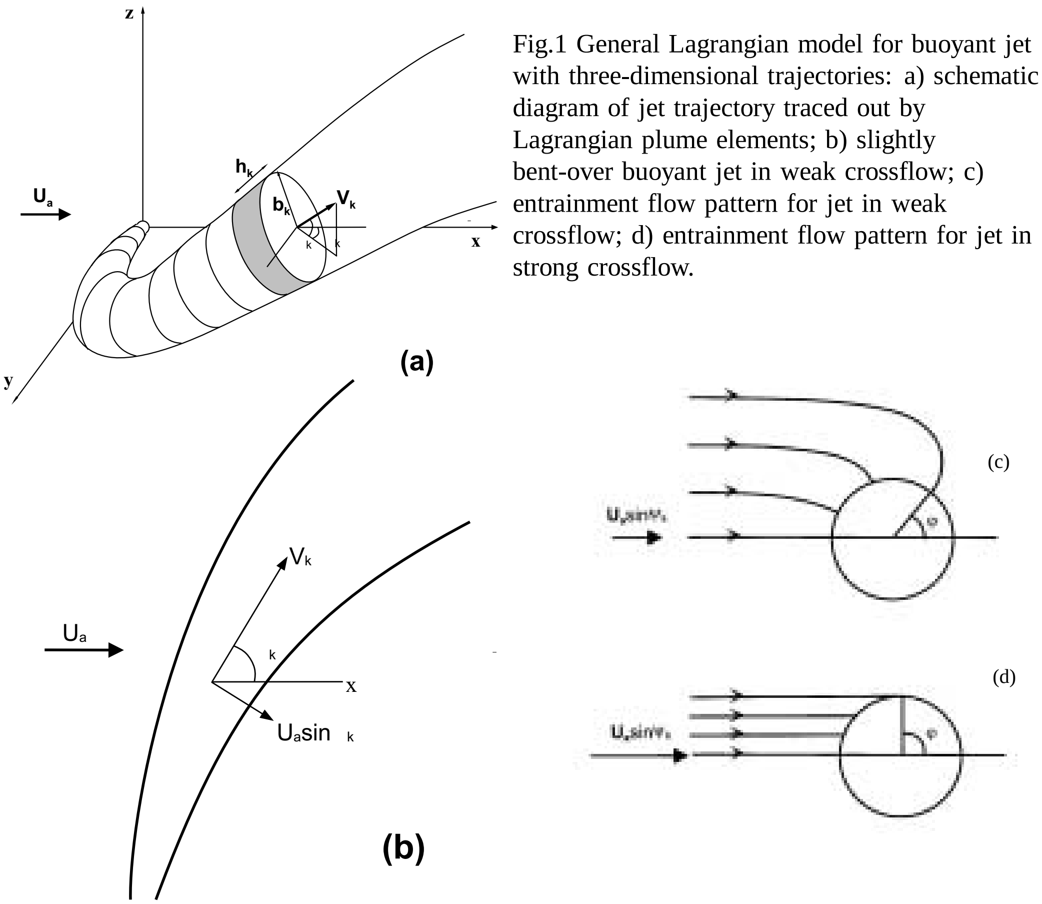

Fig.1 General Lagrangian model for buoyant jet with three-dimensional trajectories: a) schematic diagram of jet trajectory traced out by Lagrangian plume elements; b) slightly bent-over buoyant jet in weak crossflow; c) entrainment flow pattern for jet in weak crossflow; d) entrainment flow pattern for jet in strong crossflow.

LAGRANGIAN JET MODEL (JETLAG):

Overview of Model:

The Lagrangian model JETLAG (Lee and Cheung 1990) predicts the mixing of buoyant jets with three-dimensional trajectories. The model does not, strictly speaking, solve the usual Eulerian governing differential equations of fluid motion and mass transport (e.g. as adopted in Schatzmann 1981). Instead, the model simulates the key physical processes expressed by the governing equations. The unknown jet trajectory is viewed as a series of non-interfering "plume-elements" which increase in mass as a result of shear-induced entrainment (due to the jet discharge) and vortex-entrainment (due to the crossflow) - while rising by buoyant acceleration (Fig.1). The model tracks the evolution of the *average* properties of a plume element at each step by conservation of horizontal and vertical momentum, conservation of mass accounting for entrainment, and conservation of tracer mass/heat. The vortex entrainment is determined by a heuristic Projected-Area Entrainment (PAE) hypothesis originally proposed by Frick (1984) for buoyant jets with 2D trajectories, while pressure drag is ignored. Predictions of the model have compared well with basic laboratory experimental data; the model also predicts correctly the asymptotic behaviour of straight jets and plumes, and advected line puffs and thermals. Since its inception, JETLAG has been applied to many situations (e.g. Cathers and Pierson 1991; Gordon and Fagan 1991; Horton *et al.* 1997). In recent years, a better understanding of jet/plume in crossflow has been greatly facilitated by experiments using laser-induced fluorescence and digital image processing techniques, and turbulence modelling (e.g. Lee *et al.* 1996). These research findings are reflected in the current version of JETLAG. In particular, a novel treatment of the transition from the jet/plume-dominated to the ambient current-dominated regime is included. The shear entrainment formulation has also been remodelled.

Fig.1a) shows the general situation of a buoyant jet in an ambient current. Without loss of generality, we have tacitly assumed that the crossflow velocity u_a is in the +x-direction. The jet axis makes an angle of ϕ_k with the horizontal plane, and θ_k is the angle between the x-axis and the projection of the jet axis on the horizontal plane. A key component of the plume model is the computation of turbulent entrainment for the general situation of an arbitrarily-inclined plume element in a crossflow. The increase in mass of the plume element at each step, ΔM, is computed as a function of two components: the shear entrainment due to the relative velocity of the plume element and the ambient velocity in the direction of the jet axis, ΔM_s, and the vortex entrainment ("forced" entrainment) due to the ambient crossflow, ΔM_f. A maximum hypothesis, $\Delta M = Max(\Delta M_s, \Delta M_f)$, or an additive hypothesis, $\Delta M = \Delta M_s + \Delta M_f$ can be used. Comparison with basis data shows that the maximum hypothesis in general gives better results. However, the use of this hypothesis can sometimes give unreasonable predictions for a weak current. When u_a is small, e.g. around tidal slack, it is expected that shear entrainment dominates; however the relative jet velocity can decrease as the ambient current increases - thus leading to a decrease in entrainment with increasing crossflow, a result that is not borne out by experiments of plumes in weak crossflow (Lee and Cheung 1991). And yet there is hitherto no satisfactory way to handle, in a general modelling framework, the transition from the shear-induced to the forced entrainment. In addition, the limiting cases of coflowing and counterflowing jets have not been investigated in detail.

<u>*Shear entrainment:*</u>
In the current model, the shear entrainment ($\Delta M_s = E_s$) at each time step k is computed as (Fig.1):

$$\begin{aligned} E_s &= 2\pi\alpha_s b_k h_k \Delta U \Delta t && (1) \\ \alpha_s &= \sqrt{2}\,(0.057 + 0.554 sin\phi_k / F_l^2) && (2) \\ & \quad (\frac{2V_k}{\Delta U + V_k}) \end{aligned}$$

where V_k = jet velocity, $\Delta U = |V_k - u_a cos\phi_k cos\theta_k|$ is the relative jet velocity in the direction of the jet axis, and b_k, h_k are the radius and thickness of the plume element (Fig.1a); α_s and F_l are the entrainment coefficient and the local jet densimetric Froude number respectively (Lee and Cheung 1990). This new shear entrainment expression can be shown to predict correctly the limiting case of a jet in coflow (Chu *et al.* 1999); this formulation represents a significant improvement over previous work (Lee and Cheung 1990) for many flow situations.

<u>*Projected Area Entrainment (PAE)*</u>
The entrainment due to the crossflow is modelled using the Projected Area Entrainment (PAE) hypothesis (Frick 1984, Lee and Cheung 1990, Cheung and Lee 1996); this assumes that entrainment due to the crossflow (the vortex pair entrainment in the far field) is equal to the ambient flow intercepted by the 'windward' face of the plume element. In the PAE expression the incremental increase in mass at each step, $\Delta M_f (= E_f)$, can be written as:

$$\begin{aligned} \Delta M_f &= \rho_a U_a \; [2 b_k h_k \sqrt{1 - cos^2\theta_k cos^2\phi_k} \\ & \quad + \pi b_k \Delta b_k cos\phi_k cos\theta_k && (3) \\ & \quad + \frac{\pi b_k^2}{2} \Delta(cos\phi_k cos\theta_k)] \; \Delta t \end{aligned}$$

In the above, there are three contributing terms to the projected area (Eq.20 of Lee and Cheung 1990): they are respectively the projection or cylinder term E_p, a correction term due to the increase in plume width E_w, and a correction term due to plume curvature E_c; the total projected area entrainment is $E_f = E_p + E_w + E_c$.

In developing this general forced entrainment formulation, a wide variety of flow situations were considered. The formulation is generally valid without having to make special provisions - i.e. the signs and angles take care of themselves properly; it is not necessary (in fact erroneous) to force each term to be positive. A detailed discussion of the physical interpretation of the individual terms has recently been given (Cheung and Lee 1999).

The General Formulation

Consider the general situation of a jet/plume in a weak crossflow, with the jet axis making an angle of Ψ_k with the crossflow (Fig.1b); the ambient current in the plane of the jet cross-section is then $u_a sin\Psi_k$. The general entrainment formulation, which models the near-far field transition (or weak to strong current), is as follows:

$$\Delta M = E_s \frac{(\pi - \varphi_k)}{\pi} + (E_p + E_c + E_w)\ sin\varphi_k \tag{4}$$

where φ_k is a “separation angle” which delineates the relative importance of shear and vortex entrainment. φ_k is computed from the maximum radial shear entrainment velocity and the ambient velocity according to:

$$cos(\varphi_k) = Min(\frac{v_r(max)}{u_a sin\Psi_k}, 1) \tag{5}$$

For a round jet or plume in stagnant fluid, the longitudinal jet velocity is self-similar and approximately Gaussian; the radial entrainment field v_r can be obtained from continuity as a function of the local centerline velocity; since the spreading rate of straight jets and plumes is approximately the same $db/ds \approx \sqrt{2}(0.109)$, it can be shown that $v_r(max) = 0.4209\alpha_s \Delta U$, and occurs at a radial position of $r/b_k \approx \sqrt{2}$.

The above formulation is motivated by the recent work of Gaskin (1995) and Gaskin *et al.* (1995). For a jet/plume in a weak crossflow, both theory and experiments suggest the irrotational entrainment flow in the plane of the jet cross-section can be modelled as the sum of the radial entrainment flow field due to a straight jet/plume in stagnant fluid, v_r, and the uniform ambient flow defined by $u_a sin\Psi_k$. The corresponding stream function for such a flow is illustrated in Fig.1c) and Fig.1d). Eq.5 shows that as $u_a \rightarrow 0$, $\varphi \rightarrow 0$; i.e. the entrainment is entirely due to shear entrainment, while as $u_a \rightarrow \infty$, $\varphi \rightarrow \pi/2$ - i.e. entrainment is almost entirely due to crossflow (typically much greater than shear entrainment). The main consequence of this hypothesis is that for an ambient current up to a value defined by φ_k, the total entrainment is due to the shear entrainment E_s. Beyond this critical limit, the entrainment increases to slightly greater than the vortex entrainment value E_f. This hypothesis, which is supported by the recent PIV experiments of Gaskin (1995), provides a theoretical basis of modelling the weak to strong current transition (or for a given current, the transition from the jet/plume-dominated regime to the current-dominated regime) in a general framework. Note that Eq.4 and 5 are generally applicable to jets with three-dimensional trajectories.

VISUALIZATION OF MODEL PREDICTIONS: VISJET

Model Results and Discussion

Using the general formulation, the turbulent entrainment into the Lagrangian plume element can be computed at each step. The change in plume properties and the plume trajectory can then be obtained in exactly the same manner as outlined in Lee and Cheung (1990). JETLAG reproduces the correct behaviour of i) a round buoyant jet in stagnant or near stagnant fluid, and ii) an advected line puff/thermal in a bent-over momentum/buoyancy dominated jet. The current version of the model has been validated against experimental data by different investigators for: straight jets and plumes, vertical buoyant jet and dense plume in crossflow, oblique momentum jet in crossflow, horizontal buoyant jet in coflow; horizontal buoyant jet in crossflow; vertical buoyant jet in stratified crossflow; coflow and counterflowing momentum jets; buoyant plumes in weak current. We present herein only model-data comparisons for several representative cases that have until now proved difficult for integral models in general (e.g. Muellenhoff *et al.* 1985; Wood 1993). Fig.2 shows the comparison of predicted and observed trajectories and concentration of a horizontal buoyant jet in crossflow; both the jet paths in the horizontal and vertical planes and tracer concentration are well-predicted. Fig.3 shows the predicted centreline dilution for a plume as a function of $z/l_b = zu_a^3/B$, where z= elevation above source, and B=jet buoyancy flux. In a very weak current, $z/l_b \ll 1$, it is seen that the dilution is given by that of a straight plume, while significant increases in dilution can be achieved even in a weak current, $z/l_b \approx 0.1 - 1$. The present prediction represents a significant improvement over alternative formulations which do not consider the transition problem. Finally, Fig.4a) shows a comparison of predicted maximum plume rise height with data of Wright (1977), and Fig.4b) compares predicted centerline jet velocity in a counterflow with data of Chan and Lam (1998); the model predictions are in reasonably good agreement with experimental data.

Visualization of Model Predictions

VISJET is a PC-based flow visualization tool to portray clearly the evolution and interaction of multiple buoyant jets discharged at different angles to the ambient current. The system has the following features: i)*Three-dimensional graphics*: 3D color graphics is used to display the spatial layouts of all jet trajectories. When the user's virtual viewpoint or viewing direction is adjusted, the new 3D views can be displayed instantly to give the user real-time visual feedback. This feature also supports zoom-in and zoom-out to allow the user to have a close-up look at features of small scales. ii) *Animation*: The evolution of jets and other time-varying properties, such as velocity, can be displayed with special animation effects to enhance the understanding of the data displayed. The user can have a sense how the sewage discharges and evolves. iii) *Realism of ambience*: External factors, such as direction of ambient water currents and reference objects are displayed to provide a proper context for the data to be visualized. iv) *Color coding*: Color is assigned to the jet according to effluent concentration. v). *Data interrogation*: When the user wishes to know about data values defined at a point on a jet, such as velocity or concentration, this feature allows the user to locate the point of interest with a pointing device to interactively retrieve the data required. vi) *Jet inspection by intersection*: The user may use horizontal planes at different altitudes to intersect a particular jet and observe the resulting sections. This is helpful in understanding how each jet evolves when approaching the water surface. In addition, the merging of multiple plumes can be analysed in any vertical section.

Fig.5 shows images from a virtual reality simulation of the buoyant sewage plumes issuing from the proposed Hong Kong Ocean outfall diffuser risers (20 m spacing) in the form of rosette jet groups. The complex turbulent buoyant shear flow, with jet configurations ranging from a coflow jet, oblique jet, to a counterflow buoyant jet, can clearly be seen. Although the dynamic effect of jet interaction cannot be satisfactorily predicted by current models, in many situations the ports are designed to be adequately spaced apart, and treating plume merging in a kinematic manner may give reasonable results provided the individual plume trajectories and widths can be reliably predicted. The computed plume trajectories enable a meaningful definition of the initial mixing zone, and also estimates of the degree of plume merging and consequent overall dilution achieved.

CONCLUDING REMARKS

A general Lagrangian model that computes mixing of arbitrarily-inclined buoyant jets has been proposed. Model predictions are well-supported by laboratory experiments for a wide range of flow situations, including straight jets and plumes, advected line puffs and thermals, buoyant plume mixing in the near-far field transition as well as counterflowing buoyant jets. Field verification of model predictions of dilution and plume rise heights have also been reported (Horton *et al.* 1997). The model VISJET is currently being developed into an interactive modelling and educational tool (http:/www.csis.hku/∼visjet).

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REFERENCES:

[1] Cathers, B. and Peirson, W.L. (1991). “Verification of plume models applied to deepwater outfalls”, in *Proc. Int. Symp. Env. Hydr.*, Hong Kong, Lee, J.H.W. and Cheung, Y.K. (ed.), Dec.1991, Vol.1, Balkema, pp.261-266.

[2] Chan, C.H.C. and Lam, K.M. (1998). “Centerline velocity decay of a circular jet in a counterflowing stream”, *Phys. Fluids*, **10**(3), pp.637-644.

[3] Cheung, V. and Lee, J.H.W. (1996) Discussion of “Improved prediction of bending plumes”. *Journal of Hydraulic Research*, **34**(2), pp.260-262.

[4] Cheung, V. and Lee, J.H.W. (1999) Discussion of “Simulation of oil spills from underwater accidents I: model development ”, *Journal of Hydraulic Research,* **37**(3), pp.425-429.

[5] Chu, P.C.K., Lee, J.H.W., and Chu, V.H. (1999). “Spreading of a turbulent round jet in coflow”, *J. Hydr. Engr., ASCE*, **125** (2), pp.193-204.

[6] Frick, W.E. (1984) Non-empirical closure of the plume equations. *Atmospheric Environment*, **18**(4), pp.653-662.

[7] Gaskin, S.J., Papps, D.a. and Wood, I.R. (1995) “The axisymmetric equations for a buoyant jet in a crossflow”, Proc.12th Australasian Fluid Mech. Conf., Sydney, Dec.1995.

[8] Gaskin, S.J. “Single buoyant jet in a crossflow and the advected line thermal”, PhD thesis, University of Canterbury, Christchurch, New Zealand, 1995.

[9] Gordon, A.D. and Fagan, P.W. (1991). “Ocean outfall performance monitoring”, in *Proc. Int. Symp. Env. Hydr.*, Hong Kong, Lee, J.H.W. and Cheung, Y.K. (ed.), Dec.1991, Vol.1, Balkema, pp.243-248.

[10] Horton, P.R., Lee, J.H.W. and Wilson J.R. (1997). “Near-field JETLAG modelling of the Northwest New Territories Sewage Outfall, Urmston Road, Hong Kong”, *Proc. 13th Australasian Coastal and Ocean Engr. Conf., Christchurch, New Zealand*, Sept. 97, Vol.2, pp. 561-566.

[11] Lee, J.H.W. and Cheung, V. (1990) Generalized Lagrangian model for buoyant jets in current. *Journal of Environmental Engineering, ASCE*, **116**(6), pp. 1085-1105.

[12] Lee, J.H.W., Rodi, W. & Wong, C.F. (1996). ”Turbulent line momentum puffs.” *Journal of Engineering Mechanics, ASCE*, **122**(1), pp. 19-29.

[13] Muellenhoff, W.P. et al. (1985) Initial mixing characteristics of municipal ocean discharges. *Report EPA-600/3-85-073*, USEPA, Newport, Oregon.

[14] Schatzmann, M. (1981) “Mathematical modeling of submerged discharges into coastal waters,” *Proc. 19th IAHR Congress, New Delhi*, Vol. 3, pp. 239-246.

[15] Wood, I. R. (1993) “Asymptotic solutions and behaviour of outfall plumes,” *J. Hydr. Eng., ASCE*, **119**(1), pp. 555-580.

[16] Wright, S.J. (1977) ”Effects of ambient crossflow and density stratification on the characteristic behavior of round turbulent buoyant jets,” *Report No. KH-R-36,* W.M. Keck Lab. of Hydr. and Water Resour., California Inst. of Tech., Pasadena, Calif.

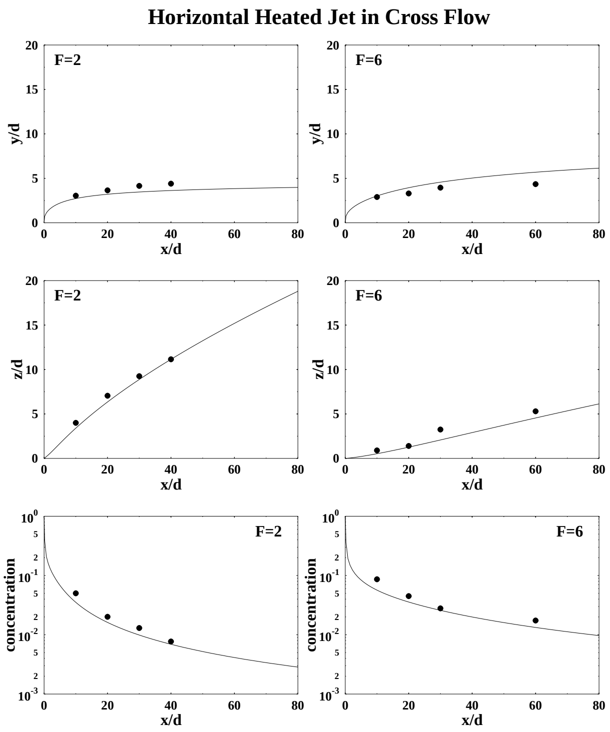


Fig.2 Comparison of predicted (solid line) jet trajectory (horizontal [x,y] and vertical [x,z] planes), and tracer concentration with data of horizontal buoyant jets in crossflow (Cheung 1991)

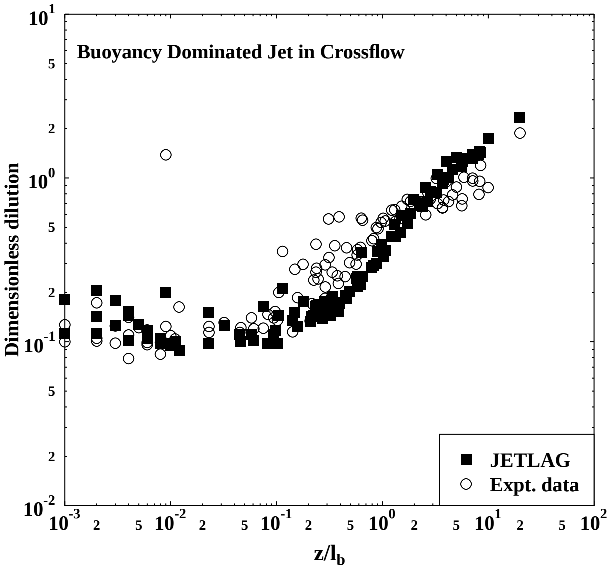
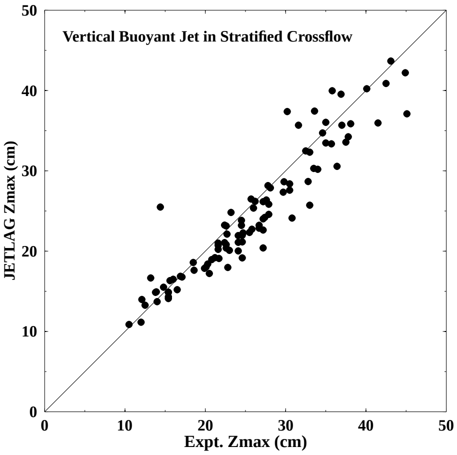
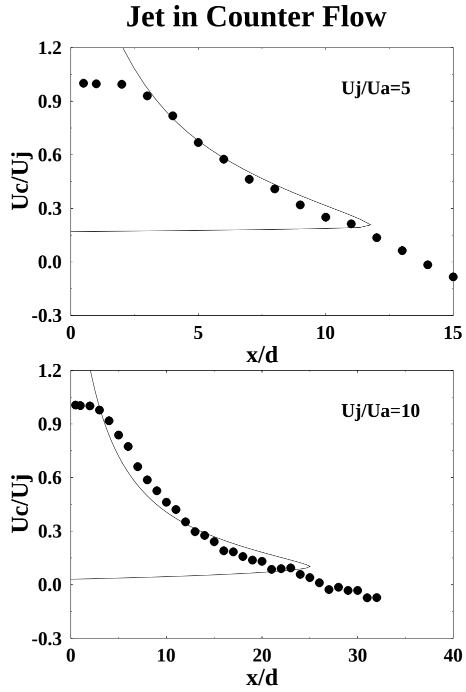


Fig.3 Comparison of predicted centerline dilution with data of buoyancy-dominated jets in weak crossflow (Lee and Cheung 1991); dimensionless jet centerline dilution is shown as a function of dimensionless depth z/l_b ($l_b = B/u_a^3$ is the buoyancy length scale, B=jet buoyancy flux)

Fig.4 Comparison of model predictions (solid line) and data (symbols) of a) maximum rise height of vertical buoyant jet in stratified fluid; and b) centerline velocity of counterflowing jet.





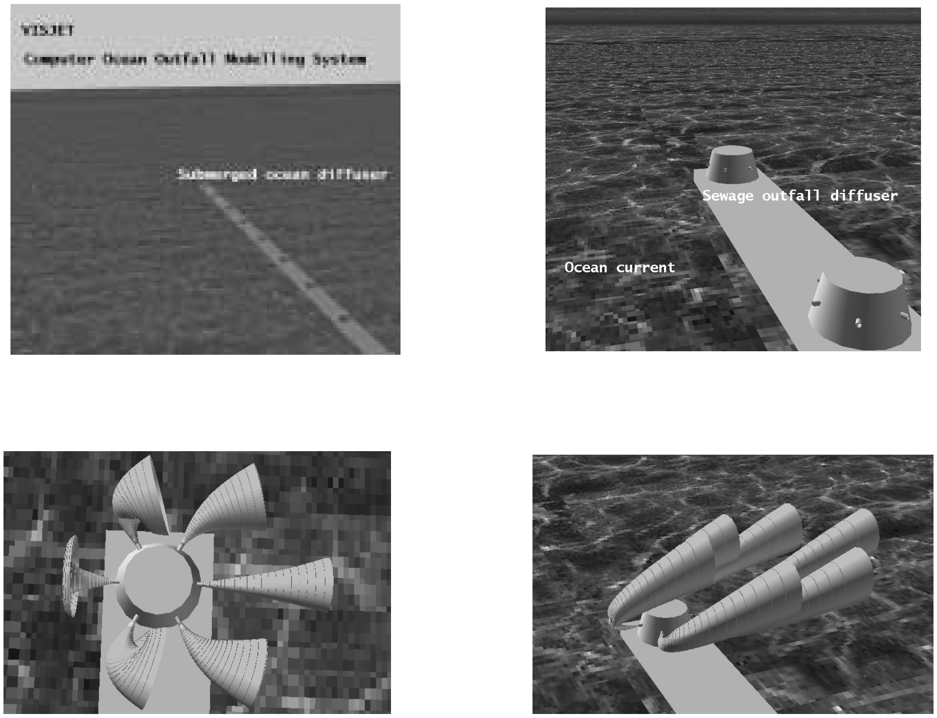


Fig.5 VISJET simulation showing merging of rosette-shaped buoyant jets above an ocean outfall riser